pressure, heat of mixing, surface tension, change of volume on mixing, and compressibility.

It has been shown that there is a strong positive deviation of the vapor pressure from Raoult's law, that the formation of mixtures is accompanied by an absorption of heat, by an expansion for mixtures rich in hexane and a contraction for those rich in aniline, that the surface tension of the mixture is less than additive, and that the compressibility of a mixture rich in hexane is greater than additive, while that of one rich in aniline is less than additive.

The theoretical bearing of the foregoing facts has been discussed briefly.

BERKELEY, CALIFORNIA.

[CONTRIBUTION FROM THE CHEMICAL LABORATORY, UNIVERSITY OF ILLINOIS.]

THE NOMON—A CALCULATING DEVICE FOR CHEMISTS.

BY HORACE G. DEMING.

Received August 7, 1917.

The slide-rule has never been in such general use among chemists as among engineers, doubtless for the reason that many of the calculations of chemistry demand a higher degree of accuracy than that obtainable with the ordinary form of this instrument. There seems to be a real demand for an instrument at once more accurate than a slide-rule and cheaper, handier, and more portable than a calculating machine.

A calculating chart designed by the writer,[1] to which the name *nomon* (*i. e.*, nomographic reckoner) has been given, seems to satisfy these requirements. It will multiply, divide, square, cube, and extract square roots and cube roots, giving results to four figures, with an average error of about one unit in the fourth place, a degree of precision about five or ten times that of an ordinary ten-inch rule. For solving proportions it is not so convenient as a slide-rule, but one should remember that in an ordinary chemical proportion two of the terms can usually be combined in a single constant, thus reducing the calculation to a simple multiplication or division.

In the construction of the nomon the logarithmic principle at the basis of the slide-rule has been abandoned entirely, and resort had to a new combination of graphical principles, the mathematical details of which will be published elsewhere. The present paper is intended merely to indicate the method of using the chart, and the special advantages that it offers in chemical calculations.

To multiply two numbers together, find one of them, neglecting decimal points, on the scale at the left of the chart, called the *principal scale* (Fig. 1). Find the other number, after dropping its first figure, on the inclined scale at the right. Connect the two points thus located by means of a

[1] Published by the University Press, Urbana, Ill.

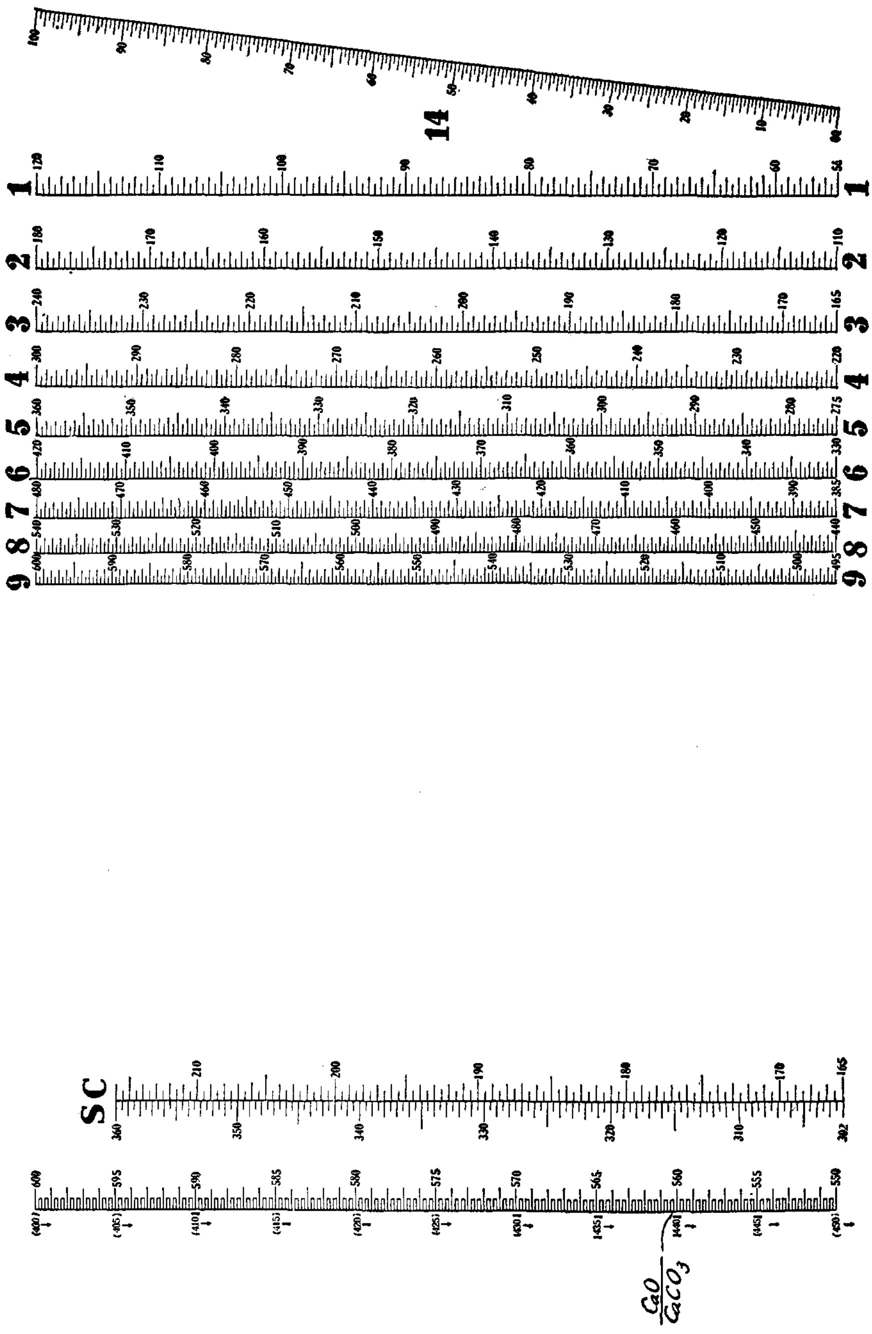

Fig. 1.—Fourteenth section of nomon, reduced from $7'' \times 10^1/_2''$.

straight line scratched on the under surface of a strip of celluloid that accompanies the chart. The product is read from the intermediate scale that is labelled above and below with the figure of the multiplier that has been dropped.

Thus the product of 586.8 and 0.7419 is found by locating the first of these numbers on the principal scale, and 41.9 on the inclined scale. If the hair-line of the transparent strip is placed to connect these two points, it will intersect the scale numbered 7 (the figure dropped from the multiplier) in the point 435.3, which is the required product. The decimal point is best located by inspection, though there is a rule for fixing its position mechanically, if desired.

Division may be performed by a process the converse of that just described for multiplication. Reciprocals may be read directly from the chart, to four, or even five figures. Auxiliary scales, placed just to the right of the principal scale, permit squares and cubes to be read directly, when points on the principal scale are connected with the point 00 of the inclined scale. Square roots and cube roots may be obtained by a converse process. Though the chart gives results directly to but four figures, its use may be combined with the ordinary methods of calculation, in such a way as to give verified products, quotients, or roots to six or seven figures, in a small fraction of the time otherwise needed.

In order to attain the degree of precision just mentioned, it has been found necessary to subdivide the principal scale, and therefore the entire chart, into eighteen sections. Thus the fourteenth section, reproduced in Fig. 1, covers the range of multiplicands from 550 to 600. A marginal index provides for instantaneous reference from one section to another. About three-fourths of the labor of disconnected multiplications and divisions falls on the last four sections.

All the scales of the chart were originally laid off on metal with a dividing engine, and all the calculations and methods of construction used in its design have been verified mechanically. In order to secure maximum legibility, special attention has been given to the method of subdividing the unit interval of each scale. In this respect the chart offers distinct advantages over the slide-rule, in which unit intervals subdivided into fifths are frequently confused by beginners with those subdivided into tenths. In most of the sections three digits of each result are given in plain figures—an additional safeguard against errors. The chart may be rolled up and carried in the pocket, and in spite of its increased precision is much less expensive than a slide-rule, the present edition selling for a dollar.

Perhaps the most important advantage of the nomon is however to be found in the fact that it may be adapted very readily to individual requirements, by inscribing special factors along the margins of the chart, beside corresponding points of the principal or inclined scales. It is thought best to leave the location of such points to the individual user, in order that the chart may not be encumbered with data of little use to him personally.

Thus points representing the common gravimetric and volumetric chemical factors, inscribed along the principal scale, may be used as constant multipliers. (The factor $CaO/CaCO_3 = 0.5603$ is indicated in Fig. 1.) A hole near one end of the transparent strip is set over a given factor and held in place with the rubber tip of a pencil, cut to a conical point. If the free end of the strip is now swung over the face of the chart, to pass through each of a series of weights or buret readings in turn, products or quotients may be read off almost as fast as an assistant can note them down. With the ordinary type of slide-rule, about half of the results of such a series of operations will lie beyond the limits of the scales, making necessary frequent resettings, if the operations are carried out in order. Or the section concerned, printed separately on bristol-board, may be fastened to the wall or table-top, and the transparent index set permanently over the given factor by means of a thumb-tack.

Construction of Special Scales.

Very frequently the special factors inscribed along the margin of the chart form a continuous scale. It is best to inscribe them along the principal scale, unless the different values used all happen to agree in their first figure, in which case the inclined scale may at times be pressed into service. If the principal scale is used, the intervals of the special scale will frequently be equal, and may be laid off with dividers. In other cases the special scale may be laid off by a method which can best be made plain by an example, which will also serve to indicate how certain complex expressions, involving even addition and subtraction, may be solved directly.

The percentage of sucrose in a sample of raw sugar or molasses is given by the expression

$$S = \frac{P}{142.66 - t/2},$$

where P is the change in rotation due to inversion of the sample under carefully prescribed conditions, and t is the temperature of polarization. Let the reciprocal of the quantity under the line of division be denoted by T. The above expression then takes the form

$$S = PT,$$

and may be solved directly, as soon as a special scale of temperatures has been marked off along one of the margins of the chart. To do this we calculate the value of $142.66 - t/2$ for three separate temperatures—say 15°, 25°, and 35°—then read the reciprocals of these quantities directly from the chart, getting the results 7399, 7683, and 7990, respectively, no attention being paid to decimal points.

We drop the initial figure 7 from each of these three numbers and mark points opposite 39.9, 68.3, and 99.0, respectively, on the inclined scale (Fig. 2). These three points are labeled with the three values of t to which they correspond. Now a most important fact about this and similar problems is that it is unnecessary to calculate the position of more than three points on the special scale to be constructed, for the reason that

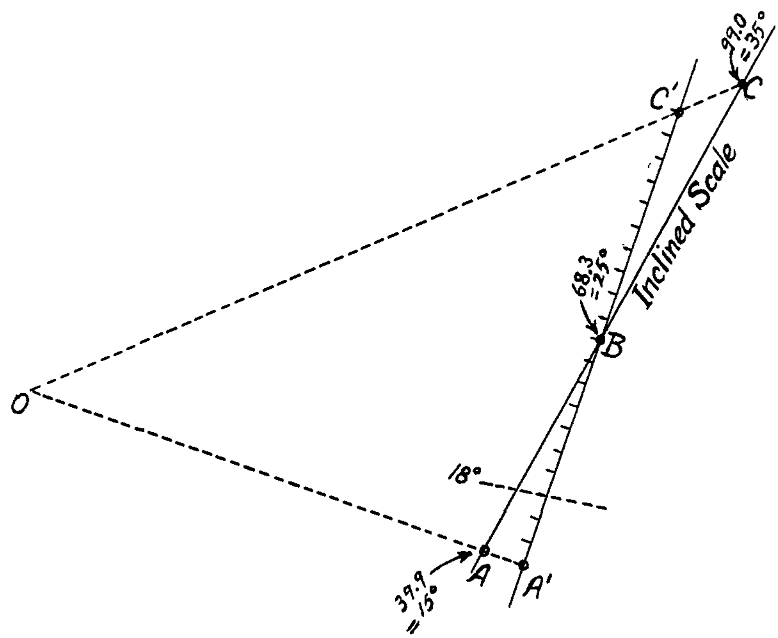

Fig. 2.—Construction of a special scale.

intermediate points may be found by a simple mechanical construction as soon as these three have been located. Such a scale is known in mathematics as a homographic, linear, or projective scale, but since few chemists are familiar with its construction, in spite of its usefulness in the graphical solution of chemical problems, it is thought well to describe the method in detail.

The construction is indicated in Fig. 2 by the dotted lines. Let A, B, and C be the three points along the inclined scale already located by direct calculation. Through one of them, most conveniently the point B, draw an auxiliary scale A' C', making any convenient angle with AC, and lay off equal graduations along it, each approximately equal to one-tenth the interval AB. Since B represents 25°, the tenth preceding point A' will represent 15°, and the tenth following point C' will represent $35°$. Through A' and C' draw straight lines passing through the corresponding points A and C already located on the scale to be constructed. These lines will intersect in a point O. From O as a center, project the graduations of the uniform scale $A'C'$. The points of intersections with AC are the required points of subdivision of the latter scale. In the figure this construction is given for $t = 18°$. As a practical suggestion, note that if the point O tends to fall outside the limits of the drawing-board, this may generally be remedied by reducing the angle between AC and $A'C'$.

Having thus constructed a scale of temperatures along the inclined scale of the chart, connect any given temperature with the point on the principal scale representing the observed change in polarization on inversion. The percentage of sucrose in the sample may then be read directly from the scale numbered 7, that being the figure dropped from each temperature factor in laying off the special scale. Of course the special scale of temperatures might also have been constructed along the principal scale of one of the sections of the chart (the sixteenth), and this is probably the best method when the change in rotation on inversion is found to vary too much to be included within the range of the principal scale of a single section.

The above method for locating intermediate points on a scale of special factors generally enables one to lay off the whole scale as quickly as half a dozen points could be found by direct calculation, and with less chance of error. The method applies with mathematical accuracy whenever a quantity T is connected with the expressed variable t by the equation

$$T = \frac{at + b}{ct + d},$$

in which a, b, c, and d are constants not all unity.

There are many expressions of this form in common use in industrial and analytical chemistry, for example those connecting the readings of the Baumé scales with specific gravity, the formula for the reduction of gas volumes to a standard temperature, those connecting the heats of dilution of certain liquids, such as sulfuric acid, with the proportion of water used, those connecting the percentage composition of compounds or mixtures with their atomic and molecular composition, and numerous others.

But even when the above relation does not hold exactly, it may be made to do so as a practical approximation by calculating a number of values of T corresponding to neighboring round values of t, then taking the points thus located in groups of three, and applying the preceding construction to locate intermediate ones. In general, points will not need to be calculated less than one inch apart, though this will depend somewhat on the function concerned, and on the degree of precision desired. Any function or expression involving but a single variable and a number of constants may thus be represented by a special scale along one of the margins of the chart, and can be made to enter a subsequent multiplication or division as if it were a simple variable. In this way the use of the nomon is extended to a class of problems very common in all branches of chemical work, to which the slide-rule is entirely unsuited.

It is not even necessary that the relation between the given function and the expressed variable should be known other than empirically. Thus a scale of specific gravities (or Brix or Baumé degrees) may be laid off along the principal scale of the corresponding section of the chart, in such a way that by connecting any specific gravity with a point on the inclined scale we may read at once the amount of the given substance in so many liters or cubic feet of solution. The special factors needed in any given case usually fall within the range of the principal scale of a single section of the chart, which may be purchased separately.

The use of a special scale in connection with the chart has a number of advantages over reference to a table. In the first place, the time needed to construct the scale is much less than that needed to calculate a table. Second, the chart is self-interpolating. Third, the calculating device thus becomes entirely self-contained, and the chance of error that occurs in carrying data from the table to a slide-rule is entirely eliminated. But perhaps most important is the fact that any change in experimental conditions generally affects all the values given in a published table and renders the latter valueless unless corrections are applied. But if a special scale, constructed for certain conditions, is merely penciled in, it may be erased in a moment and replaced by a new one. For example, the inversion constant in the preceding illustration is altogether dependent on the method of inversion, and it is impossible to draw up a table that would suit all conditions.

As a further example, consider the determination of nitrogen by the Kjeldahl method. Here the formula is

$$x = 100\ nf/w,$$

where x is the required percentage of nitrogen, n is the difference between the number of cubic centimeters of alkali used in the titration of the sample and that required in a blank determination, f is the equivalent of each cubic centimeter of the alkali in grams of nitrogen, and w is the weight of the sample. As long as the given stock of reagents lasts, f will be a constant, hence the quantity $100\,f/w$, which we shall call W, may be treated as a simple function of the weight of sample taken. The formula then reduces to

$$x = nW.$$

If a scale of weights is now laid off along the principal scale, opposite points corresponding to the calculated values of W, we need only connect the point representing the given weight with the point on the inclined scale representing the difference between the titration figures for blank and sample (after dropping the first figure of this difference). The percentage of nitrogen is then read from one of the nine numbered scales. It is possible to convert any of these scales into a double scale (like the scale S–C) so that the percentage of nitrogen and the corresponding percentage of protein may be read from the chart at the same instant.

When the stock of reagents runs out, or the value of the factor f changes, one need only erase the scale of weights penciled along the left-hand margin of the chart, and put in a new scale. This takes but a few moments, for the weight of sample used in routine work is apt to vary between quite narrow limits, or may even be constant.

It is possible to extend the principle that has just been described to the construction of special scales that take two variables into account. But because the details of this would require considerable space, they will be reserved for a manual that is being prepared to accompany future editions of the chart. The nomon may thus be used for the direct solution of expressions of the form

$$x = f(u, v).f(w),$$

in which $f(u, v)$ is any function of the two variables u and v, and $f(w)$ is any function of the variable w. Expressions of this form are of very frequent occurrence in chemical and engineering practice. The nomon furnishes an accurate, simple, and elegant method for solving any one of them in an instant, as soon as the special scales, needed have been constructed. Indeed, the value of the chart as an aid to calculation is apt to be limited only by the user's own ingenuity and mathematical intuition, since a necessary preliminary to the application of nomographic methods is the recognition of the types of calculations to which such methods apply.

Summary.

The use of the nomon is described. It is a calculating chart that multiplies, divides, squares, cubes, and extracts square roots and cube roots, giving four figures, with an average error of about one unit in the fourth place. A method is indicated, by means of which any expression involving but one or two variables, with a number of constants, may be represented by a special scale along one of the margins of the chart, to be treated thereafter as a simple variable, subject to multiplication and division in the usual way.

URBANA. ILL.